Characterization of Anisotropic Substrate Materials for Microwave Applications

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Abstract

This paper deals with the characterization of uniaxially anisotropic substrates. On the basis of the measured effective permittivity of a microstripline the permittivity tensor of the substrate is calculated using an optimization routine and a Spectral-Domain Analysis (SDA) for the microstripline properties.

Introduction

As most of the substrates for high-T_c superconducting film deposition are uniaxially anisotropic it is necessary to take this property into account when simulating circuits. This means on the one hand that the analysis program must be able to handle anisotropic materials, and on the other hand that the permittivity tensor of the used substrate must be known exactly. The determination of permittivity tensors of anisotropic substrates for integrated circuits is performed on the basis of the measured phase constant of a single microstrip line at several frequencies. The investigations have shown that the dispersion characteristics of microstrip lines on isotropic and anisotropic substrate materials, respectively, differ in a manner which allows to determine even the permittivity tensor of the used substrate material.

Theory

Anisotropic Substrate Materials in Spectral-Domain Analysis

The calculations are based on a Spectral-Domain Analysis using the Galerkin method [1, 2, 3]. For this purpose the Dyadic Greens functions are calculated for biaxially anisotropic substrate materials on the basis of the time dependent Maxwell equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{1}$$

which are treated for the case of a dielectric anisotropy.

$$\vec{D} = \varepsilon_0 \cdot \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \cdot \vec{E} \tag{2}$$

All field components are transformed to the spectral domain, e. g.

$$\tilde{E}_x = \int_{-\infty}^{+\infty} E_x \cdot e^{jk_x x}\, dx \tag{3}$$

The evaluation of the equations (1) and (2) leads to some coupled differential equations

$$\eta_{xx} \cdot \tilde{E}_x + \eta_{xy} \cdot \tilde{E}_y = -\frac{\partial^2 \tilde{E}_x}{\partial z^2} \tag{4a}$$

$$\eta_{yx} \cdot \tilde{E}_x + \eta_{yy} \cdot \tilde{E}_y = -\frac{\partial^2 \tilde{E}_y}{\partial z^2} \tag{4b}$$

which can be decoupled [4]using a coordinate transformation to an $\hat{x}\hat{y}$-system. The solutions of (4a) and (4b) are found to be:

$$\tilde{E}_{\hat{x}} = \tilde{E}_{\hat{x}1} \cdot e^{+j\sqrt{\lambda_1}\, z} + \tilde{E}_{\hat{x}2} \cdot e^{-j\sqrt{\lambda_1}\, z} \tag{5a}$$

$$\tilde{E}_{\hat{y}} = \tilde{E}_{\hat{y}1} \cdot e^{+j\sqrt{\lambda_2}\, z} + \tilde{E}_{\hat{y}2} \cdot e^{-j\sqrt{\lambda_2}\, z} \tag{5b}$$

with appropriate values for λ_1 and λ_2:

$$\lambda_1 = \frac{\eta_{xx} + \eta_{yy}}{2} + \sqrt{\left(\frac{\eta_{xx} - \eta_{yy}}{2}\right)^2 + \eta_{xy}\eta_{yx}} \tag{6a}$$

$$\lambda_2 = \frac{\eta_{xx} + \eta_{yy}}{2} - \sqrt{\left(\frac{\eta_{xx} - \eta_{yy}}{2}\right)^2 + \eta_{xy}\eta_{yx}} \tag{6b}$$

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These solutions can be transformed to the original coordinate system and the boundary conditions at the dielectric/vacuum interface can be evaluated. This leads to the Dyadic Greens functions as they are required for the Spectral-Domain Analysis.

$$\tilde{E}_x = \tilde{G}_{E_x,J_x} \cdot \tilde{J}_x + \tilde{G}_{E_x,J_y} \cdot \tilde{J}_y \qquad (7a)$$

$$\tilde{E}_y = \tilde{G}_{E_y,J_x} \cdot \tilde{J}_x + \tilde{G}_{E_y,J_y} \cdot \tilde{J}_y \qquad (7b)$$

Dispersion Characteristics of Microstriplines on Anisotropic Substrates

Using the Dyadic Greens functions as calculated above, the dispersion characteristics of some microstriplines as shown in figure 1 on isotropic and anisotropic substrates have been determined with a Spectral-Domain Analysis.

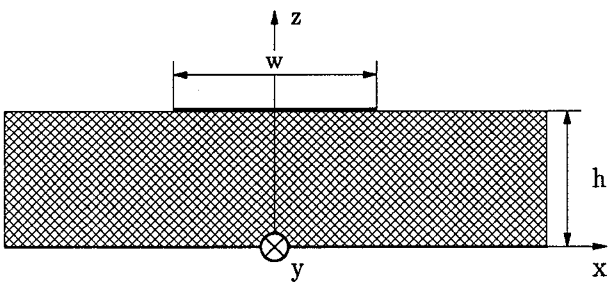

Figure 1: Investigated microstripline

Doing this one finds out that the curves differ in a way which promises the possibility to characterize the used substrates anisotropy on the basis of the measured effective permittivity of a single microstrip line, see figure 2.
The solid curve shows ε_{eff} for a linewidth of 600 μm on a 400 μm thick sapphire substrate with $\varepsilon_{\|} = \varepsilon_{zz} = 11.60$ and $\varepsilon_{\perp} = \varepsilon_{xx} = \varepsilon_{yy} = 9.40$. The dotted curve is based on the data of the same line on a substrate with $\varepsilon_{\text{eq,iso}} = 11.24$.
When analyzing those calculated curves one notices that the results obtained from those data are unique.

Error Analysis

To estimate the errors which occur when data are disturbed, one can take the results of a calculation and add an artificial error to this data. Doing this one finds out that the value of $\varepsilon_{\|}$ is weakly affected, but the value of $\varepsilon_{\perp}$ is much stronger changed, as it is shown in table 1. The reason for this behaviour is that the field

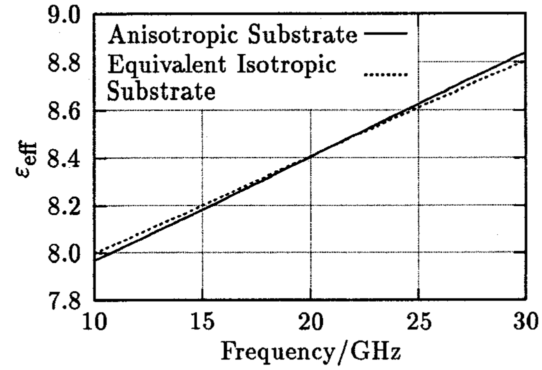

Figure 2: Microstripline on anisotropic and equivalent isotropic substrate

Art. error	$\varepsilon_{\perp}$	$\varepsilon_{\|}$	$\varepsilon_{eq,iso}$
0%	9.40	11.60	11.24
1%	8.45	11.89	11.36
2%	8.67	11.97	11.47

Table 1: Analysis of disturbed data

components perpendicular to the optical axis nearly vanish for the case of a microstripline.

Results

Analysis of a Sapphire Substrate

Two microstriplines with different widths on a sapphire substrate have been investigated. The effective permittivity of both lines is shown in figure 3 in the frequency range from 10 to 30 GHz.
Both substrates were 400 μm thick. It was assumed that the c-axis of the substrate was parallel to the z-axis (see figure 1), but the optimization led to the result that it was parallel to the x-axis. So it became necessary to recalculate the line properties on the basis of the following ε-tensor:

$$\varepsilon_0 \cdot \begin{pmatrix} \varepsilon_{\|} & 0 & 0 \\ 0 & \varepsilon_{\perp} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{pmatrix} \qquad (8)$$

Table 2 shows the results of the optimization.
The results of either line were taken to calculate the dispersion of the other line. The obtained curves are

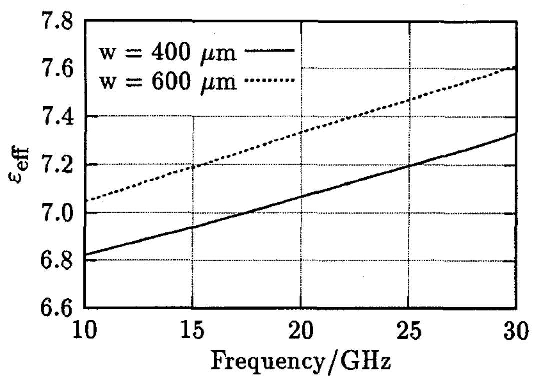

Figure 3: Measured data

w/μm	Pts.	$\varepsilon_{\parallel}$	$\varepsilon_{\perp}$
400	9	12.32	9.36
400	21	12.36	9.35
600	9	11.85	9.46
600	21	11.87	9.46

Table 2: Results of optimization (Pts. means number of frequencies

shown in figures 4 and 5. The errors of both calculations are within a 0.5% range concerning ε_{eff}.

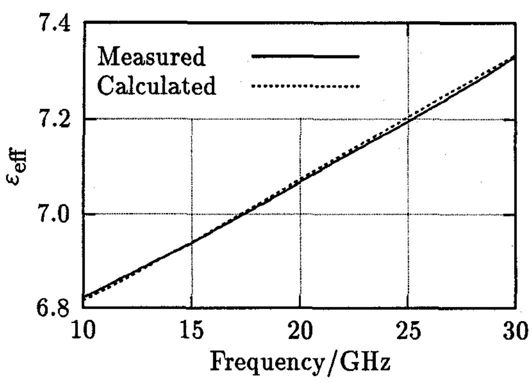

Figure 4: Comparison of measured and calculated data for the 400 μm line

A comparison between the results obtained in this work and results reported in the literature [5, 6], which are $\varepsilon_{\parallel}$ = 11.60 and $\varepsilon_{\perp}$ = 9.40, shows an error of 0.56% and 0.64%, respectively, for $\varepsilon_{\perp}$, and

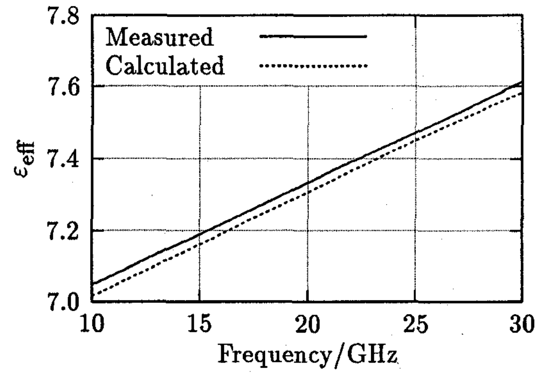

Figure 5: Comparison of measured and calculated data for the 600 μm line

6.55% and 2.33% for $\varepsilon_{\parallel}$. This quite large difference for $\varepsilon_{\parallel}$ can be explained with the weak electrical field in the x-direction.
In order to improve the results for $\varepsilon_{\parallel}$ it is planned to investigate either a coplanar line or the odd–mode of some coupled microstrips as there is a quite strong electrical field in x-direction in both cases.

Analysis of an Alumina Substrate

Another investigated structure was a microstrip line on an alumina substrate with ε_r = 9.8 and h = 250 μm. Trying to fit the measured curve, one finds out that $\varepsilon_{r,iso}$ should be 9.825. But as it is easily seen in figure 6, the two curves are not really identical.

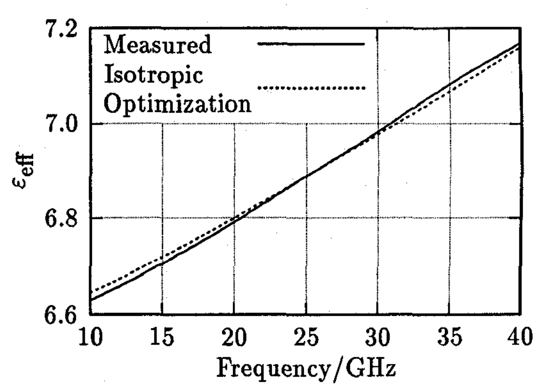

Figure 6: Comparison of measured and calculated data (isotropic) for a microstripline on an alumina substrate (w = 250 μm)

Another optimization, allowing a uniaxial anisotropic character of the substrate, led to the result that the alumina substrate is anisotropic, as mentioned in [7], with $\varepsilon_{\perp} = 8.607$ and $\varepsilon_{\|} = 10.159$, showing a much better conformity of the curves (see figure 7).

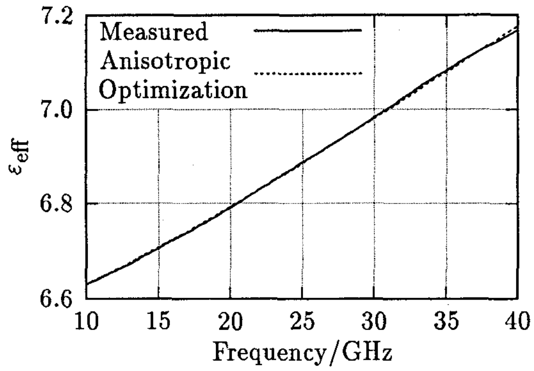

Figure 7: Comparison of measured and calculated data (anisotropic) for a microstripline on an alumina substrate (w = 250 μm)

In order to prove these results, another, wider, microstrip line was investigated. This led to a very good agreement concerning $\varepsilon_{\|}$, as the value 10.133 was found now. A much poorer agreement was reached with $\varepsilon_{\perp}$, being 7.664 in this case, which again is a consequence of the vanishing electrical field in the x- and y-direction.
The results are shown in figures 8 and 9.

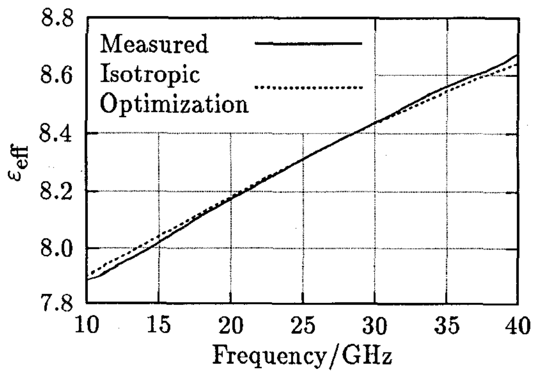

Figure 8: Comparison of measured and calculated data (anisotropic) for a microstripline on an alumina substrate (w = 1040 μm)

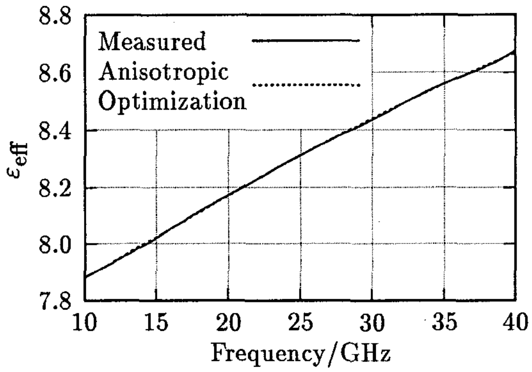

Figure 9: Comparison of measured and calculated data (anisotropic) for a microstripline on an alumina substrate (w = 1040 μm)

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